

# Incorporating Operational and Economic Uncertainties in Acquisition Decisions for a Logistics Vehicle Fleet

**Dr. Yaw Asiedu**

Defence R&D Canada

Centre for Operational Research and Analysis

101 Colonel By Drive

Ottawa, ON

CANADA K1A 0K2

[yaw.asiedu@forces.gc.ca](mailto:yaw.asiedu@forces.gc.ca)

## **ABSTRACT**

*The first three phases (design, acquisition and use) of the life cycle of military equipment often span multiple years. Because of the lengths of these phases, the economic and operational environments assumed during the design and acquisition decision process may not be present when the system is delivered or during its use. Variations in economic parameters can lead to cost overruns, and variations in operational environments can lead to system specifications, deemed appropriate during the earlier phases, being inadequate for actual operational requirements. These uncertainties should therefore be accounted for during the design and/or acquisition process.*

*This paper discusses how Chance-Constrained Goal Programming can be employed to determine the optimal procurement decision to satisfy operational demands while minimizing the risk of a budget overrun. A model is developed for the selection of an optimal vehicle fleet mix for an operational scenario where the commodity demands and vehicle costs are assumed to be normally distributed. The aim was to determine the optimal fleet capable of satisfying the demands with a given probability while minimizing the potential budget overrun at a given probability level, that is, the Value at Risk (VaR). Three different probability levels (90%, 95% and 99%) for the satisfaction of demand are considered for a VaR at a 95% level.*

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## **1.0 INTRODUCTION**

Logistic support vehicles like most military systems are used for extended periods and are expensive to acquire and operate. The procurement of such systems is a very important process since the wrong system would saddle the military organisation with a non-performing asset for an extended period. This can adversely impact operations and defence budgets. However, procuring the most cost effective system is complicated by the fact that procurement decisions are made well in advance of the (unknown) operations to which the system will be deployed, and also by the fact that military operations are extremely diverse.



Planning scenarios (or simply, scenarios) are often employed to describe a representative spectrum of operations in which military forces may be called upon to act. They provide the context to assess tasks which must be done and the capabilities that may be required to undertake each task. Later in the force development process, appropriate platforms or systems are assessed to identify those that best fulfil the capability mix of requirements.

Given that a scenario provides the contest and setting for possible future operations, data on parameters such as the level of demand and system cost, cannot be known with certainty. Also, the economic and operational environments assumed during the design and acquisition decision processes may not be the same as when the system is acquired or fielded. Furthermore, suppliers may come from different countries thus exposing the process to currency risks. The risk of exceeding an allocated budget or cost target, or of not being able to satisfy actual operational demands in full should therefore be accounted for in the decision making process by incorporating the uncertainties in the cost and operational parameters.

Scenarios employed in the context of logistics vehicle fleet mix analysis invariably involve the task of delivering a set of supplies to a single base or multiple bases that are geographically dispersed. The problem is often modelled as a loading problem (single base) or a vehicle routing problem (multiple bases) and the minimum cost fleet mix that can deliver the supplies determined.

Variants of the vehicle fleet mix problem have been well studied both in the broader scientific community and within Defence Research and Development Canada Centre for Operational Research and Analysis (DRDC CORA). In the open literature, recent research have focused on applying various metaheuristics (such as genetic algorithms, simulated annealing, tabu search, etc.) to determine optimal fleet mixes [1–3]. However, the issue of uncertainty in demand and cost have not be addressed adequately either internally or in the open scientific literature. Interested readers may consult [4] for a more extensive literature survey on fleet composition and routing.

Within DRDC, the Directorate Materiel Group Operational Research - Acquisition Support Team (DMGOR AST) has conducted a number of studies in support of the Medium Support Vehicle System (MSVS) project [5, 6] and Logistics Vehicle Modernisation Project (LVMP) [7]. In these studies, requirements were assumed to be equivalent to the amount of supplies that were consumed daily. Mixed integer programming models were then used to determine the fleet mix that minimized the logistics footprint required to replenish base supplies daily. The fleet mix was therefore optimized for the delivery of that deterministic specification of demand of supplies. Asiedu and Hill [7] incorporated the dynamic aspects of the problem by considering daily changes in demand due to backorders (previous demand that were not completely fulfilled) but did not treat uncertainties in the demand.

There are a number of ways to account for uncertainties in a fleet mix optimization model. A common approach is to determine the fleet mix using the mean values of the uncertain parameters. The solution may then be adjusted upwards by a predetermined factor to account for the possibility of having to satisfy demands in excess of the mean values. Such solutions have high associated risks. Another approach is to consider the worst case scenario and determine the optimal fleet for that case. While this ensures that the demand would always be satisfied, the cost of the fleet may be exorbitant and may exceed the allocated budget.

A better approach to handling uncertainties is the use of stochastic programming techniques such as Chance-Constrained Goal Programming (CCGP). The advantage of using stochastic programming over conventional deterministic programming is that the uncertainty is explicitly incorporated into the solution.

## 1.1 Aim

This paper proposes a CCGP model to determine the optimal vehicle fleet mix for replenishment scenarios where the commodity demands and vehicle costs are uncertain. The intent is to determine a fleet capable of totally satisfying demands with a given probability while minimizing the Value at Risk<sup>1</sup> (VaR) at a given probability level. To illustrate how different probability levels impact the fleet selection and associated VaR, the approach is applied to a sample scenario where the demand and vehicle unit costs are assumed to be independent and normally distributed.

## 1.2 Scope

The deterministic equivalent of a CCGP model is a mixed integer non-linear programming model that may be non-convex. These types of problems are very difficult to solve. This paper does not attempt to develop a solution methodology for the model but rather, solves it using AlphaECP. This is a software package based on the Extended Cutting Plane (ECP) algorithm [8] for solving both convex and non-convex, mixed integer non-linear programming models. The solver is freely available through the Network-Enabled Optimization System (NEOS) Server [9, 10]. Furthermore, the cost and demand distributions used in this paper are subjective and not based on statistical analysis of historical data.

## 1.3 Paper Organization

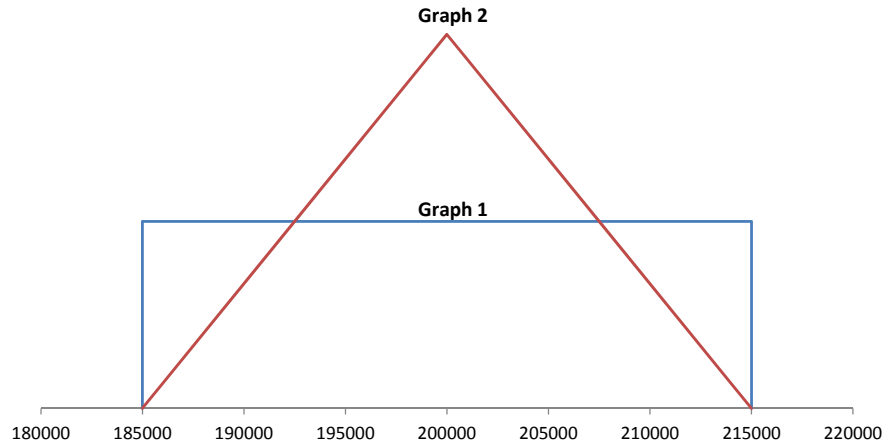
The remainder of this paper is organized as follows. The next section provides a brief discussion on how uncertainties in model parameters may be handled. A more detailed discussion of stochastic programming, Chance-Constraint Programming (CCP) and Goal Programming (GP) is contained in Annex A. Section 3 contains the description of the operational scenario employed in the sample problem with the associated mathematical model discussed in Annex B. The results from the optimization model are discussed in Section 4 and concluding remarks presented in Section 5.

## 2.0 HANDLING PARAMETER UNCERTAINTY

The majority of optimization applications in practice employ deterministic models where (design, operational, cost, etc.) parameters are assumed to be known with certainty. However, this may not be the case in reality. As indicated earlier, in the case of identifying the optimal fleet mix of a logistics support system, the actual demands and system costs may not be known with certainty. Assuming otherwise may result in a fleet that is inappropriate for the intended application. The parameters may therefore be more accurately represented as random variables. For example, as illustrated in Figure 1, the cost of a truck could be assumed to be uniformly distributed between \$185,000 and \$215,000 (Graph 1) or triangularly distributed with the same range and a mean of \$200,000 (Graph 2).

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<sup>1</sup>Given  $0 < \alpha < 0.5$  and  $\beta = 1 - \alpha$ , VaR is defined as a value such that the probability that the loss/cost over a specified time exceeds the value is equal to  $\alpha$ . That is, VaR is the  $100\beta$ -th percentile of the loss/cost probability distribution function. Consequently, a VaR value may be denoted as either  $VaR_\alpha$  or  $VaR_\beta$ . Since by convention  $\alpha < 0.5 < \beta$ , it should be obvious which notation is being used in a given context.



*Figure 1: Sample representations of the uncertain cost of a truck.*

## 2.1 Stochastic Programming Methods

Incorporating parameter uncertainties in a model presents computational and analytical challenges since all possible outcomes of the uncertain parameters must be taken into account in making the optimal decision. The two main methods that have been proposed to address uncertainty in optimization problems are stochastic programming [11, 12] and robust optimization [13, 14].

Robust optimization attempts to generate a solution that would be feasible for the worst case realization of the uncertain parameters as defined by the decision maker. This is more suitable for situations where the ranges for the uncertain parameters are known but the distributions may not be known.

On the other hand, stochastic programming is based on the notion that it may be impossible to find a set of system specifications that would meet all future operational needs. In some instances, the unexpected operational need can be met by some recourse actions taken once the need is known. For example, contracting for private strategic lift capability is an option for a military faced with a higher than expected lift capability requirement.

The use of recourse action may not be an option in some applications or a cost cannot be assigned to the recourse action in a reasonable manner. For example, unlike strategic lift requirements, tactical lift requirements may not be contracted out for security and operational reasons. In such circumstances, one would rather insist on decisions guaranteed to meet operational requirements ‘as much as possible’. This approach is known as probabilistic programming or CCP [12].

Chance-Constrained Programming allows a decision maker to specify the level of acceptable risk for operational and design requirements (constraints) when model parameters are uncertain. Such constraints differ from conventional constraints in the sense that they only have to be satisfied a certain percentage of the time. This percentage is in essence, the confidence the decision maker has that an acceptable solution would satisfy the constraint when the uncertain parameters become known.

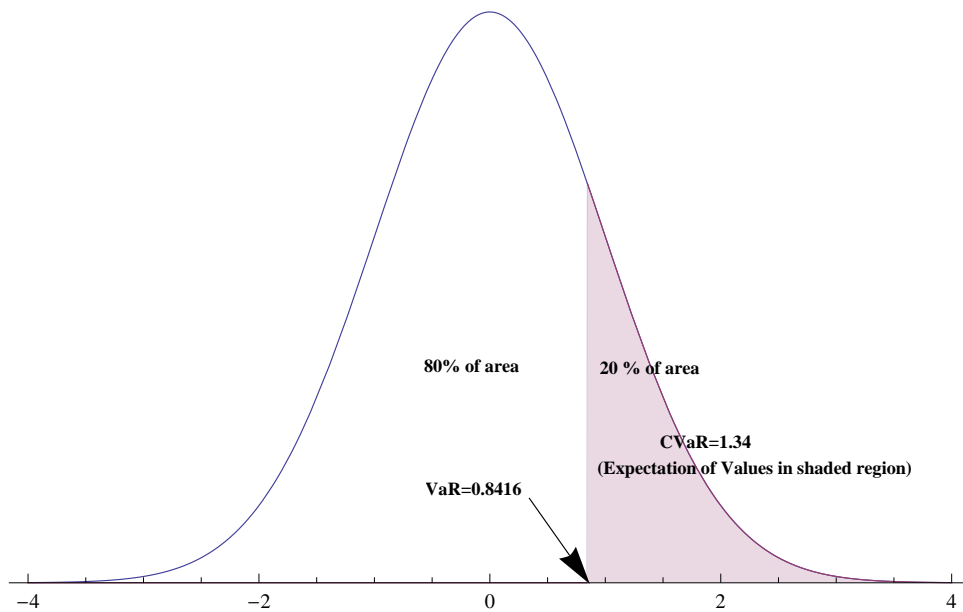
## 2.2 Uncertainty in Objective Function

A mathematical programming technique known as Goal Programming may be used to convert the objective (function) of a problem into a constraint by specifying a target value for the objective and the deviation from

this value specified as the new objective to be minimized.

Using CCGP (the combination of CCP and GP) permits the minimum deviation to be determined at a given probability level. When the objective function represents a loss function and its parameters are uncertain, the deviation is equivalent to the risk metric, VaR (see footnote 1 for definition). In the case of system acquisition, the loss function may be viewed as the total cost of the acquisition or the expenditure beyond an approved budget and the ideal value set to zero or the allocated budget, respectively.

The VaR is the value that may be exceeded with a given probability. However, it does not provide any information on the loss once this value is exceeded. A second and better risk metric, with respect to system optimization, is the Conditional Value at Risk (CVaR) [15]. CVaR is the expectation of the loss function values that exceed the VaR. Figure 2 illustrates the difference between the two metrics using the standard normal distribution and a level of 0.8 or 80% (note that the  $VaR_{\beta}$  notation is used throughout this report).



*Figure 2: Illustration of the concepts of VaR and CVaR at the 0.8 level.*

All the uncertain parameters in this report are assumed to be normally distributed. Consequently, using either VaR or CVaR as the objective will produce the same optimal decision (see [15]). VaR is used for the model discussed in this report because of the relationship between VaR constraints, CCP and GP. Interested readers may review the material in Annex A for a more detailed discussion of stochastic programming, CCP, GP, VaR and CVaR.

### 3.0 SCENARIO DESCRIPTION AND MATHEMATICAL MODEL

In order to determine the most appropriate logistics vehicle fleet, various scenarios have to be analyzed and the final fleet determined by the judicious combination of the results from each scenario. Scenarios used for this purpose may have several bases geographically dispersed ([5], [6]), or a single base ([5]). When support to multiple bases is considered, the determination of the optimal fleet of vehicles can be formulated as a variant

of the Vehicle Routing Problem (VRP). In the case of a single base, the problem is simply a container loading and assignment problem.

The VRP has been treated extensively in the open literature (the review of Laporte and Osman [16] contains over 500 papers) and also in previous DRDC studies ([5], [6] and [7]). This study therefore focuses on the issue of making optimal fleet mix decisions in the presence of data uncertainty and thus employs a scenario with a single base supported by a depot. The problem is presented in the form of a scenario in this section while the mathematical formulation is discussed in Annex B.

### 3.1 The Sample Scenario

The force in the scenario is a task group of approximately 2800 personnel deployed at a single base and supported from a single depot. This number was chosen to ensure that changes in the probability level for the demand constraint would result in some changes in the optimal fleet. On each day, a single trip is organized from the depot to deliver commodities based on the supply requests received. The amount of each commodity demanded is uncertain but assumed to follow a known distribution. The objective is to determine the number and types of vehicles, trailers and containers in the fleet required to supply the troops. In order to reduce cost, the intention is not to specify a fleet that is capable of supplying the demand in all cases, but rather one that can ensure that the demand can be fully met a certain percentage of the time, and partially the rest of the time. That is, demand can be completely satisfied with a given probability.

The commodity types and amounts that have to be delivered to the base are shown in Table 1 together with the maximum quantity that may be loaded onto standard North Atlantic Treaty Organization (NATO) pallets. The weight of a pallet of ammunition can vary significantly from 100 kg to over 1800 kg [17]. For example, the weight for 155 mm artillery is about 1,200 kg and 1,620 kg for .50 calibre machine guns. A NATO standard unit load has a mass of up to 1130 kg (2500 lb) [17]. This amount was used in this study.

The mean values for the demand in Table 1 are based on standard planning consumption rates and the number of personnel—2800. With the exception of ammunition, the daily consumption rates are based on forecasted standard usage rates indicated in the Staff Data Handbook [18]. The ammunition rate is based on data from [5]. As stated above, the actual demand is uncertain. It was assumed that the demand is distributed normally with a standard deviation equal to 15% of the mean. These are also shown in Table 1. This is just an assumption and is not based on any statistical analysis of historical data.

*Table 1: Standard parameters for the commodities.*

Group	Identifier	Class Name	Weight per Pallet (kg)	Usage Rates (kg/man/day)	Demand (kg)	
					Mean	Std. Dev.
Rations	CT1	Individual Meal Package	698	4.98	13944	2091.6
	CT2	Meals Ready to Eat	1006	2.4	6720	1008
	CT3	Fresh Rations	350	3.1	8680	1302
	CT4	Water (Personal Consumption)	900	13.97	39116	5867.4
POL	CT5	Packaged POL	425	1.4	2800	420
General Stores	CT6	Engineering Stores (Construction)	544	6.3	17640	2646
	CT7	Defence Stores (Barrier)	288	4.3	12040	1806
	CT8	General and Technical	300	2.9	8120	1218
Ammo	CT9	Ammunition	1130	7	19600	2940

The use of personnel-based consumption rates is a simplification of how resources are actually used. The consumption rate for some commodities may not be directly proportional to the personnel strength; packaged petroleum, oil and lubricants (POL) is more likely to be correlated with the number of vehicles at the base.

Commodities are grouped into four compatibility groups defined by NATO. These are: rations (individual meal packs (IMP) and water), packaged POL, ammunition and general stores (note that not all classes of mixed stores identified by NATO are considered in this paper). Items in different compatibility groups may be combined on a carrier provided they are packed in different containers.

### 3.2 Logistics Fleet Options

To transport supplies, they are first loaded onto pallets which are in turn loaded directly onto cargo trucks or into containers (20 ft containers, quadcons or bicons) which are then transported on palletized load system/load handling system (PLS/LHS) vehicles. The types of trucks, trailers and containers considered in the study are discussed below. Similar to the demand, all the cost parameters are assumed to be normally distributed but with a standard deviation equal to 10% of the mean.

#### 3.2.1 Truck Options

The characteristics of the trucks under consideration to lift the daily supplies are shown in Table 2. These are the PLS/LHS (TK1-TK3) and cargo (TK4-TK7) variants of trucks ranging in payload from 4 to 20 tonnes. A truck with a PLS/LHS has the capability to load, transport and unload standardized containers. With the exception of TK7 which has a 10 ft bed, all trucks have a 20 ft bed.

*Table 2: Characteristics of the available trucks.*

Identifier	Truck Name	Payload (kg)	Bed Length(ft)	Bulk Capacity	Type	Towing Capacity(kg)	Cost (\$000)	
							Mean	Std. Dev.
TK1	8 tonne	8000	20	4	PLS/LHS	8000	400	40
TK2	16 tonne	16000	20	4	PLS/LHS	16000	600	60
TK3	20 tonne	20000	20	4	PLS/LHS	16000	700	70
TK4	8 tonne cargo	8000	20	4	Cargo	8000	300	30
TK5	16 tonne cargo	16000	20	4	Cargo	16000	500	50
TK6	20 tonne cargo	20000	20	4	Cargo	16000	600	60
TK7	4 tonne 10 ft cargo	4000	10	2	Cargo	8000	220	22

Each truck has a bulk capacity corresponding to the number of quadcons it can carry. For cargo trucks which cannot transport containers, this number is the same as a PLS/LHS truck with the same truck bed length. For example, since a 20 ft cargo truck can carry four quadcons, it will have a bulk capacity of four.

The mean and standard deviations of the cost of the trucks are also shown in Table 2. The cost of the trucks increase with increasing capacity and for a given size, the cargo variant is cheaper.

#### 3.2.2 Trailer Options

Similar to the trucks, there are PLS/LHS (TR1-TR3) and cargo (TR4-TR6) variants as shown in Table 3, and each trailer has a bulk capacity corresponding to the number of quadcons it can carry. In this study, it is assumed that PLS/LHS trucks can tow trailers TR1-TR3, while cargo trucks can only tow trailers TR4-TR6. Trailers TR2 and TR5 are phantom trailers used in the mathematical model to represent the case where a 16 tonne trailer

is used but cannot be loaded to its full capacity because it is towed by a truck with a lower towing capacity. In this instance, the capacity is restricted to 8 tonnes. The cost of the 8 tonne trailers are about \$5,000 lower than the 16 tonne trailer for both the cargo and PLS/LHS variants.

*Table 3: Characteristics of the available trailers.*

Identifier	Trailer Name	Payload (kg)	Bed Length (ft)	Bulk Capacity (kg)	Type	Cost (\$000)	
						Mean	Std. Dev.
TR1	8 tonne	8000	20	4	PLS/LHS	90	9.0
TR2	16 tonne RL	8000	20	4	PLS/LHS	95	9.5
TR3	16 tonne	16000	20	4	PLS/LHS	95	9.5
TR4	8 tonne cargo	8000	20	4	Cargo	90	9.0
TR5	16 tonne cargo RL	8000	20	4	Cargo	95	9.5
TR6	16 tonne cargo	16000	20	4	Cargo	95	9.5

### 3.2.3 Container Options

The characteristics of the containers considered in this report are shown in Table 4. Containers come in many sizes and types, but only certain sizes are standardized to facilitate intermodal transportation. Those considered in this report are the standard 20 ft containers, bicons and quadcons. Bicons are roughly half the size of a 20 ft container and a quadcon is roughly a quarter of the size. Two bicons, four quadcons or one bicon and two quadcons can be linked together to form one twenty-foot equivalent unit (TEU), and handled exactly as a standard 20 ft container. The cost of a container is effectively independent of the size or type.

*Table 4: Characteristics of the available containers.*

Identifier	Container Type	Bulk Capacity	Payload (kg)	Tare Weight (kg)	Footprint	Cost	
						Mean	Std. Dev.
CO1	Quadcon	4	4280	800	1	4000	400
CO2	Bicon	8	8560	1600	2	4000	400
CO3	20 ft	20	20000	2700	4	4000	400
CO4	20 ft cargo	10	-	0	4	-	-
CO5	10 ft cargo	10	-	0	2	-	-

To facilitate the application of the model to a wide variety of vehicles and container combinations, it is assumed that all vehicles, including cargo vehicles, transport goods in a container. It is further assumed that the truck bed of cargo vehicles is a weightless phantom container with an infinite payload. This assumption has no effect since the total weight of the contents of the phantom container is limited to that of the vehicle. Note that these containers also have no cost assigned to them.

Containers are assumed to have a footprint corresponding to its quadcon equivalence. That is, the footprint of a container is the number of quadcons that would have to be linked together to create an equivalent container. For example, the footprint of a bicon is two and that for a 20 ft container is four. The 20 ft container cannot be transported by vehicles with 10 ft beds, and only cargo vehicles can transport the phantom cargo containers (CO4 and CO5).



## 4.0 RESULTS AND DISCUSSIONS

As stated previously, determining the optimal fleet mix to deliver goods to a single base location is simply a container loading and assignment problem. This is a difficult problem to solve but is made easier in this study by assuming that standard NATO pallets are used with the number of pallets that each container can transport, predetermined. This eliminates the need to explicitly solve the loading problem.

The intent in using a scenario approach in a fleet mix analysis is often to identify the type of vehicles and containers that an organisation would need and/or the relative proportions of the vehicle and container types in the final fleet. The actual fleet acquired may therefore be based on the judicious combination of the results from all the scenarios considered. It is assumed in this study that only the scenario discussed earlier is being used to make the decision and that the types of vehicles and containers identified would be acquired in the proportion specified by the optimisation model. The objective function can thus be stated simply as the sum of the cost of the vehicles and containers selected in the scenario.

The problem was modelled as a CCGP and solved using AlphaECP [8], a solver available through the NEO Server [9, 10]. AlphaECP was selected because it was the only solver available through the NEO Server that could solve the problem. The complete mathematical model of the problem is discussed in more detail in Annex B.

To simplify the model, it was assumed that there was a fixed number of each vehicle and container type available. The assumed maximum number available for each truck, trailer and container type is shown in Tables 5, 6 and 7, respectively. These numbers were identified through trial and error so that the size of the model was as small as possible and no solutions required the maximum number available in the optimal fleet. The value of  $B$  was set to zero and that of  $U$ , set to four. That is, no budget was assumed and at most, only four different types of trucks may be in the fleet.

To assess how different probability levels affect the fleet mix, the model was solved with an objective function level of 95% (i.e.,  $\beta_o = 0.95$  and  $\text{VaR}_{0.95}$  (equation B22) is what is minimized) for demand constraint levels of 90%, 95% and 99% (i.e.,  $\beta_c = 0.9, 0.95$  and  $0.99$ , respectively). Multiple values of  $\beta_o$  were not considered for specific values of  $\beta_c$  because, for objective function coefficients that are normally distributed, the optimal fleets would be the same. In the discussion below, the 90% case is used as the baseline or reference for comparison.

*Table 5: Assumed maximum number of each truck type available.*

Truck	Maximum Available
8 tonne	4
16 tonne	4
20 tonne	1
8 tonne cargo	6
16 tonne cargo	1
20 tonne cargo	1
4 tonne 10 ft cargo	3



Table 6: Assumed maximum number of each trailer type available.

Trailer	Maximum Available
8 tonne	4
16 tonne RL	2
16 tonne	4
8 tonne cargo	6
16 tonne cargo RL	2
16 tonne cargo	1

Table 7: Assumed maximum number of each container type available.

Container	Maximum Available
Quadcon	6
Bicon	5
20 ft	12

#### 4.1 Fleet Mix

Tables 8, 9 and 10 show the trucks, trailers, and containers in the optimal fleet mix, respectively. In the aforementioned tables, the 20 ft cargo and 10 ft cargo phantom containers are excluded. These containers were abstract modelling constructs. Using the 90% case as an example, if the fleet mix specified is purchased, then it would be enough to supply the demanded commodities 90% of the time. For the remaining 10% of the time, the fleet would not be enough to meet all demands but would still be enough to supply at least 90% of the needed amounts.

With respect to the trucks, the smaller trucks are favoured. The 16- and 20-tonne cargo trucks and the 20 tonne trucks are never used. The trailer fleets are quite similar for the three cases with the main difference between them being the use of an additional 8 tonne trailer for the 99% case. Even though for the 95% case, a 16 tonne cargo RL trailer is used in lieu of an 8 tonne cargo trailer. Recall that the 16 tonne cargo RL is similar to the 8 tonne cargo trailer with respect to the payload.

Unlike the trucks, containers with higher bulk capacities and payloads are used. This may be a reflection of the fact that the cost is not dependent on the size of the container. This makes the use of say, a 20 ft container preferable to the use of either four quadcons or two bicons in most cases.

Table 11 shows the total fleet payload and bulk capacity for the three cases. As expected, the total payload increases with increasing demand probability level. It is seen that a 5.6% relative change in level (90% to 95%) resulted in only a 2.2% change in payload and no change in bulk capacity. The corresponding values for a change from 90% to 99% are 16% and 14% for payload and bulk capacity, respectively. These are higher than the change in level, 10%.

*Table 8: Number of each truck type in final fleet for a various demand constraint probability levels.*

Truck Type	Demand Probability Level (%)		
	90	95	99
8 tonne	2	2	3
16 tonne	3	3	3
20 tonne	0	0	0
8 tonne cargo	3	5	5
16 tonne cargo	0	0	0
20 tonne cargo	0	0	0
4 tonne 10 ft cargo	2	0	0

*Table 9: Number of each trailer type in final fleet for various demand constraint probability levels.*

Trailer Type	Demand Probability Level (%)		
	90	95	99
8 tonne	2	2	3
16 tonne RL	0	0	0
16 tonne	3	3	3
8 tonne cargo	5	4	5
16 tonne cargo RL	0	1	0
16 tonne cargo	0	0	0

*Table 10: Number of each container type in final fleet for demand constraint probability levels.*

Container Type	Demand Probability Level (%)		
	90	95	99
Quadcon	2	4	0
Bicon	1	0	2
20 ft	9	9	11



Table 11: Total effective payload and bulk capacity for the fleet for various demand constraint probability levels.

Demand Probability Level (%)	Payload		Bulk Capacity	
	(kg)	(%) Change	No. of Pallets <sup>a</sup>	(%) Change
90	369,620	-	296	-
95	377,620	+2.2	296	0
99	428,220	+16	336	+14

<sup>a</sup> The sum of the bulk capacities of containers and cargo vehicles.

**4.2 Cost Risk Measures**

Figure 3 shows the acquisition cost distributions for the three cases. That is, for each fleet mix specified in Tables 8, 9 and 10, the actual cost could be any value in the range specified by the corresponding probability distribution. These distributions were developed using equation B1 and the cost parameters from Tables 2, 3 and 4.

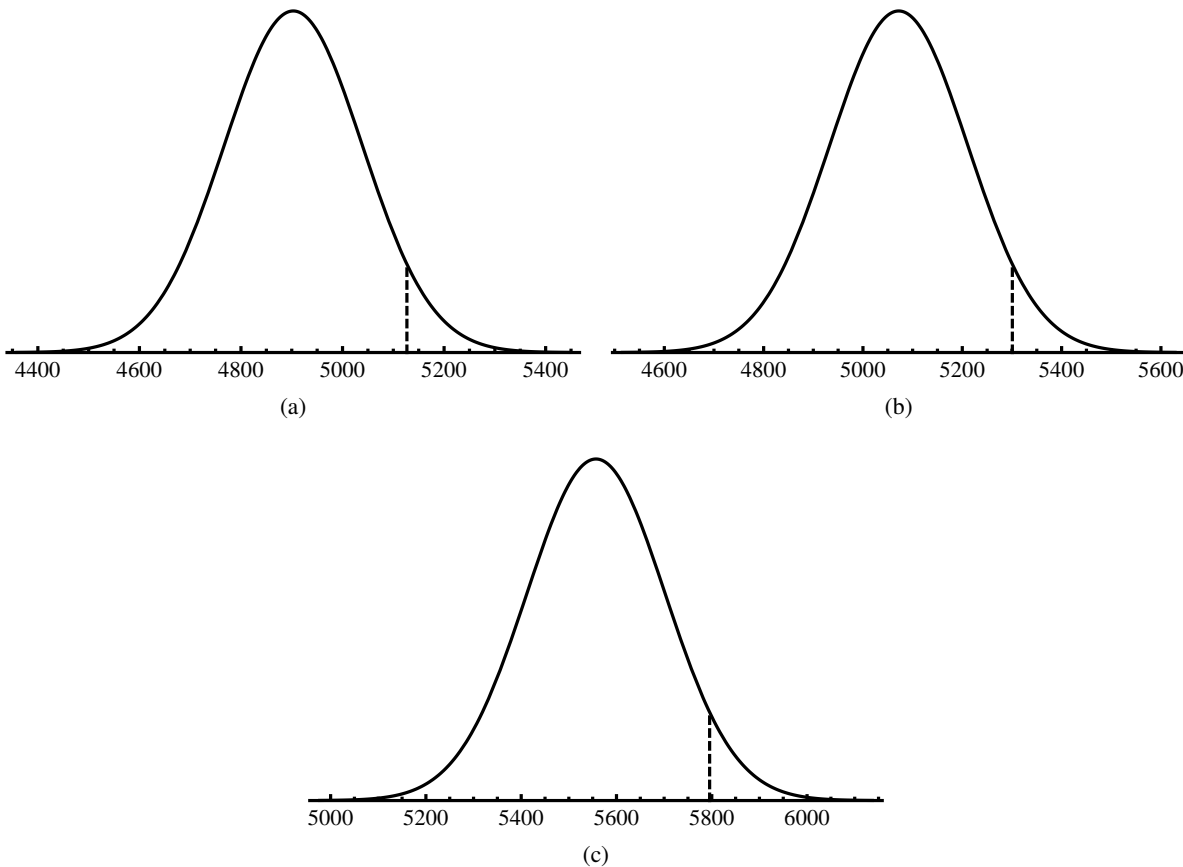


Figure 3: Total cost distributions and VaR<sub>0.95</sub> for demand constraint probability levels of 90% (a), 95% (b) and 99% (c) with cost values in thousands of dollars.

The  $VaR_{0.95}$  values are indicated by the vertical lines in Figure 3. The exact values are shown in Table 12 together with the values for  $CVaR_{0.95}$ . The  $VaR_{0.95}$  values are from solving the model while the  $CVaR_{0.95}$  values were calculated using equation A10. However, the  $CVaR_{0.95}$  values are not shown in Figure 3 since they correspond to points at the extreme right tail of the distributions. Note that  $CVaR_{0.95}$  could have been optimized directly by using an objective which is the sum of equations B22 and A10 (or the equivalent equation for the specific distribution), in the model in Section B3.

As expected,  $VaR_{0.95}$  and  $CVaR_{0.95}$  increase with increasing demand probability level. The change in  $VaR_{0.95}$  and  $CVaR_{0.95}$  are 3.4% and 3.3% for the 95% demand probability level and, 13.0% and 12.7% for the 99% demand probability level. Similar to the payload and bulk capacity values, these values show that a change in demand probability level from 90% to 99% requires disproportionately more money to be risked while a change from 90% to 95% requires relatively less money to be risked.

*Table 12: Optimal risk measures for various demand constraint probability levels.*

Demand Probability Level (%)	$VaR_{0.95}$		$CVaR_{0.95}$	
	(\$ 000)	% Change	(\$ 000)	% Change
90	5,126	-	5,407	-
95	5,300	+3.4	5,588	+3.3
99	5,795	+13	6,095	+12.7

$VaR$  and  $CVaR$  can be used for planning in a multitude of ways. Two approaches are discussed next using the results for the 95% case. Table 12 shows that  $VaR_{0.95}$  is equal to \$5.3 M. This implies that, if the organization decides to acquire the fleet as specified in Tables 8, 9 and 10, then there is only a 5% chance of the actual price exceeding \$5.3 M. The organization can therefore specify a budget equal to this amount and a contingency fund of \$288,000 ( $CVaR_{0.95} - VaR_{0.95}$ ). This translates to a contingency of 5.4% and almost 100% (99.99%) confidence that there will be enough funds to cover the cost. Effectively, the organization is assured that there will be money for the acquisition assuming the uncertainties in the parameters have been correctly identified.

In the case where a budget has already been allocated, that is  $B$  in equation B23 is set,  $VaR$  represents the cost overrun at the specified level. For illustrative purposes, if it is assumed that  $B = \$5$  M, then the  $VaR_{0.95}$  and  $CVaR_{0.95}$  will be \$300,000 and \$588,000 for the 95% case, respectively. In this case, the organization may specify a contingency equal to  $VaR_{0.95}$ . Consequently, there is a 95% chance that the contingency amount would be enough to address a project cost that exceeds the allocated budget. The organization could also buy insurance to cover the amount of \$288,000 ( $CVaR_{0.95} - VaR_{0.95}$ ), the expectation (mean) for the 5% of cases where the cost overrun would be in excess of the contingency fund.

As noted above, these are just two examples of how the outputs of the model can provide insights into budgetary requirements.

## 5.0 CONCLUDING REMARKS

Procurement decisions are often made with uncertain data. These data uncertainties can impact the choices made and the operational utility of the procured system. They should therefore be addressed adequately during the decision process. A CCGP model was developed to illustrate how operational and economic uncertainties can be included in optimal procurement decisions.

Using CCGP allows the decision maker to specify the cost and operational performance risk levels that are acceptable to the organisation and determine the optimal decisions accordingly. Where the levels are not



known, different performance and cost levels can be considered to permit cost-performance trade-off analysis to be conducted to determine the most appropriate levels.

The utility of the CCGP model was illustrated by applying it to the selection of an optimal logistics vehicle fleet mix. A sample scenario where the demand and vehicle unit costs were assumed to be normally distributed was used in the application. Three different probability levels (90%, 95% and 99%) for the satisfaction of demand were considered for a 95% level for the cost. Using the 90% case as reference, it was observed that a change in demand probability level from 90% to 99% requires disproportionately more money to be risked while a change from 90% to 95% requires relatively less money to be risked.

Although the modelling approach presented was illustrated with a logistics vehicle fleet mix problem, the approach (and not necessarily the model) is applicable to other procurement problems where there are uncertainties and an appropriate balance between the cost risk and performance or operational risk is sought.

## ABOUT THE AUTHOR

Dr. Yaw Asiedu is a defence scientist at the Defence Research and Development Canada Centre for Operational Research and Analysis (DRDC CORA). He is currently the head of the Acquisition Support Team.

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## Annex A STOCHASTIC PROGRAMMING TECHNIQUES

### A1 Introduction

A linear optimization problem can be stated as:

$$\min \sum_{j=1}^n c_j x_j, \quad (A1)$$

$$\text{subject to: } \sum_{j=1}^n a_{ij} x_j \geq b_i, \quad i = 1, 2, \dots, m, \quad (A2)$$

$$x_j \geq 0,$$

where  $c_j$  are the  $n$  objective function coefficients,  $x_j$  are the  $n$  decision variables,  $a_{ij}$  are the  $m \cdot n$  constraint coefficients and  $b_i$  are the  $m$  constants.

The majority of optimization applications in practice employ deterministic models where  $c_j$ ,  $a_{ij}$  and  $b_i$  are assumed to be known with certainty. However, this may not be the case in reality. The parameters  $c_j$ ,  $a_{ij}$  and  $b_i$ , may therefore be more accurately represented as random variables.

The two main methods that have been proposed to address uncertainty in optimization problems are stochastic programming [11, 12] and robust optimization [13, 14]. Robust optimization attempts to generate a solution that would be feasible for the worst case realization of the uncertain parameters as defined by the decision maker.

On the other hand, stochastic programming is based on the notion that it may be impossible to find a set of values of the decision variables that would exclude later constraint violations caused by unexpected random effects. Stochastic programming models can be further grouped into recourse models [19] and, probabilistic programming or CCP [12]. Recourse models correct constraint violations by taking some recourse actions, at additional cost, once the violation is observed. CCP models try to identify decisions guaranteeing feasibility 'as much as possible' [20]. Only the CCP modelling approach is discussed in the remainder of this annex.

### A2 Chance-Constrained Programming

Chance-constrained programming was developed by Charnes and Cooper [12] as a means of describing constraints in terms of attainment probabilities or acceptable risks. If  $0 < \beta < 1$  is the degree of confidence desired by the decision maker, then  $\alpha = 1 - \beta$  is the risk acceptable to the decision maker. Given  $\beta$ , the CCP equivalent of the constraints defined by equation A2 may be specified as:

$$P \left\{ \sum_{j=1}^n a_{ij} x_j \geq b_i \right\} \geq \beta_i, \quad i = 1, 2, \dots, m \quad (A3)$$

or

$$P \left\{ \sum_{j=1}^n a_{ij} x_j \geq b_i, i = 1, 2, \dots, m \right\} \geq \beta, \quad (A4)$$

where  $P$  is a probability measure. Equation A3 implies that each individual constraint may be violated but only at most  $(100 \times \alpha_i)\%$  of the time. Similarly, the joint constraint defined by equation A4 may be violated but

only at most  $(100 \times \alpha)\%$  of the time. Such constraints differ from conventional constraints in the sense that they only have to be satisfied a certain percentage of the time.

The constraints defined by equation A3 are the unconditional or individual chance constraints while equation A4 defines a combined or joint chance constraint. Equation A3 considers the probability to satisfy each one of the constraints independently from the others while equation A4 is the probability to simultaneously satisfy all the constraints. The model in this report is based on individual chance constraints. Consequently, only that is discussed further in this annex.

### A3 Deterministic Equivalent Model

In the general case, it is assumed that  $c_j$ ,  $a_{ij}$  and  $b_i$  are all random variables. Approaches for solving CCP models first converts them to deterministic equivalent models which are then solved using well established deterministic solution techniques. The following sections discuss how to derive the deterministic equivalent of a CCP model.

#### A3.1 The Deterministic Equivalent Constraint

With respect to the constraints, three cases can arise: random  $a_{ij}$  and deterministic  $b_i$ , deterministic  $a_{ij}$  and random  $b_i$ , and random  $a_{ij}$  and  $b_i$ . The deterministic equivalent for each case under the assumption that the random variables are normally distributed is as follows:

1. *Random  $a_{ij}$*

$$\mathbb{E}\left[\sum_{j=1}^n a_{ij}x_j\right] - Q_{\beta_i}\sqrt{\mathbf{X}^T\Phi_i\mathbf{X}} \geq b_i \quad (\text{A5})$$

where,  $\mathbf{X} = (x_1, x_2, \dots, x_n)$ ,  $\Phi_i$  is the  $i^{\text{th}}$  covariance matrix and  $Q_{\beta_i}$  is the  $\beta_i$ -percentile of the standard normal density function;

2. *Random  $b_i$*

$$\sum_{j=1}^n a_{ij}x_j \geq \mathbb{E}[b_i] + Q_{\beta_i}\sqrt{\mathbb{V}[b_i]}; \quad (\text{A6})$$

3. *Random  $a_{ij}$  and  $b_i$*

$$\mathbb{E}\left[\sum_{j=1}^n a_{ij}x_j - b_i\right] - Q_{\beta_i}\sqrt{\mathbf{X}^T\Phi_i\mathbf{X} - \mathbb{V}[b_i]} \geq 0. \quad (\text{A7})$$

When  $a_{ij}$  are independent, equation A5 reduces to

$$\mathbb{E}\left[\sum_{j=1}^n a_{ij}x_j\right] - Q_{\beta_i}\sqrt{\mathbb{V}\left[\sum_{j=1}^n a_{ij}x_j\right]} \geq b_i \quad (\text{A8})$$

and equation A7 reduces to

$$\mathbb{E}\left[\sum_{j=1}^n a_{ij}x_j - b_i\right] + Q_{\beta_i}\sqrt{\mathbb{V}\left[\sum_{j=1}^n a_{ij}x_j - b_i\right]} \geq 0. \quad (\text{A9})$$

### A3.2 The Deterministic Equivalent Objective Function

There are a number of ways that objective function uncertainties can be handled in optimization models. Charnes and Cooper [12] suggested three approaches (assuming a maximisation problem): maximise the mean (M model), minimize variance (V model) and maximise the probability (P Model). A similar approach known as the mean-variance framework was proposed for portfolio optimization in the seminal work of Markowitz [21]. However, this framework was found to have shortcomings in credit portfolio optimizations [22]. VaR was thus suggested in its stead for some financial applications.

The use of VaR as a risk metric was spurred by amendments to the Basel Accord in 1996 to require banks to use it and also to set aside capital for meeting market risk [23]. In the financial industry,  $VaR_{\beta}$  is defined as the value such that the probability that the loss/cost over a specified time exceeds is  $\alpha = 1 - \beta$ . The value of  $\alpha$  is often take as 0.01 or 0.05. The same VaR value may be denoted as  $VaR_{\alpha}$ . One of the shortcomings of using VaR as a risk metric is that it does not measure the extent of the losses beyond the VaR. Also, it is difficult to optimize VaR numerically when the objective function is not normally distributed. Rockafellar and Uryasev [15] introduced the Conditional Value at Risk (CVaR) to address these and other concerns. CVaR, also known as expected shortfall, mean shortfall, mean excess loss or tail VaR, is the expectation of the loss function values that exceed VaR (see Figure 2 for an illustration of the difference between VaR and CVaR).

For a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ , the CVaR at a confidence of  $\beta$  ( $CVaR_{\beta}$ ) can be calculated using [24]

$$CVaR_{\beta} = \mu + \frac{\exp(-0.5q^2(\beta))}{\sqrt{2\pi\alpha}} \sigma . \quad (A10)$$

Expressions for calculating CVaR for other distributions can be found in [25]. As a metric in optimization, CVaR has superior properties in many respects compared to VaR (details can be found in [22], [26] and [27]). However, for normal or elliptical distributions, VaR, CVaR, or minimum variance will produce the same optimal solution (see [15]). An approach to optimize CVaR and determine VaR simultaneously for general distributions is presented in [15]. However, calculation of the CVaR is approximated using simulated data.

Goal programming is a multi-objective programming technique first developed by Charnes, Cooper and Ferguson [28]. In GP, goals are set for each objective and a solution that minimizes the deviations from the goals is determined. In stochastic GP (goal programming with uncertainties in the objective function coefficients or targets), an objective function can be converted to an equivalent constraint and deterministic objective. For example, equation A1 can be changed to:

$$\min \delta^+ , \quad (A11)$$

$$\text{subject to: } P \left\{ \sum_{j=1}^n c_j x_j - \delta^+ \leq g \right\} \geq \beta , \quad (A12)$$

$$\delta^+ \geq 0 , \quad (A13)$$

where  $g$  is the aspiration (the goal) and  $\delta^+$  is the positive deviation from the goal. When equation A1 represents a loss function,  $\delta^+$  is equivalent to the VaR. In the case of system acquisition, the loss function may be viewed as the total cost of the acquisition or the expenditure beyond an approved budget.

### A3.3 Chance-Constrained Goal Programming (CCGP)

From the foregoing discussions, given a general linear multi-objective optimization problem,

$$\begin{aligned}
 & \max \sum_{j=1}^n c_{lj}x_j, & l = 1, 2, \dots, \tilde{t}, \\
 & \min \sum_{j=1}^n c_{lj}x_j, & l = \tilde{t} + 1, \tilde{t} + 2, \dots, t, \\
 & \text{subject to:} & \\
 & P\left\{ \sum_{j=1}^n a_{ij}x_j \leq b_i \right\} \geq \beta_i, & i = 1, 2, \dots, \tilde{m}, \\
 & P\left\{ \sum_{j=1}^n a_{ij}x_j \geq b_i \right\} \geq \beta_i, & i = \tilde{m} + 1, \tilde{m} + 2, \dots, m, \\
 & x_j \geq 0
 \end{aligned} \tag{A14}$$

where  $c_{lj}$ ,  $a_{ij}$  and  $b_i$  are independent random variables, the deterministic nonlinear programming equivalent of the model can be obtained as:

$$\begin{aligned}
 & \min \sum_{l=1}^{\tilde{t}} \delta_l^- + \sum_{l=\tilde{t}+1}^t \delta_l^+, \\
 & \text{subject to:} \\
 & \mathbb{E} \left[ g_l - \sum_{j=1}^n a_{ij}x_j \right] + Q_{\beta_l} \sqrt{\mathbb{V} \left[ g_l - \sum_{j=1}^n a_{ij}x_j \right]} \leq \delta_l^- & l = 1, 2, \dots, \tilde{t}, \\
 & \mathbb{E} \left[ \sum_{j=1}^n a_{ij}x_j - g_l \right] + Q_{\beta_l} \sqrt{\mathbb{V} \left[ \sum_{j=1}^n a_{ij}x_j - g_l \right]} \leq \delta_l^+ & l = \tilde{t} + 1, \tilde{t} + 2, \dots, t, \\
 & \mathbb{E} \left[ \sum_{j=1}^n a_{ij}x_j - b_i \right] + Q_{\beta_i} \sqrt{\mathbb{V} \left[ \sum_{j=1}^n a_{ij}x_j - b_i \right]} \leq 0 & i = 1, 2, \dots, \tilde{m}, \\
 & \mathbb{E} \left[ \sum_{j=1}^n a_{ij}x_j - b_i \right] - Q_{\beta_i} \sqrt{\mathbb{V} \left[ \sum_{j=1}^n a_{ij}x_j - b_i \right]} \geq 0 & i = \tilde{m} + 1, \tilde{m} + 2, \dots, m, \\
 & x_j, g_l, \delta_l^+, \delta_l^- \geq 0.
 \end{aligned}$$

## Annex B THE OPTIMIZATION MODEL

The detailed mathematical model of the problem described in Section 3.0 is presented in this section. The deterministic mixed integer programming model of the problem is presented first before the deterministic equivalent model of the CCGP variant of the problem is presented.

The complete notation for the model is summarized below. When required, indices of parameters (known model input values) are indicated in parenthesis, e.g.  $c(m)$ , and indices of variables (unknown model outputs) are indicated as subscripts. Note that a number of variables used in Annex A are redefined here.

*Inputs-Sets:*

$S$	Trucks (may contain more than one of each truck type)
$R$	Trailers (may contain more than one of each trailer type)
$J$	Vehicles, $J = S \cup R$
$M$	Containers (may contain more than one of each container type)
$I$	Commodities
$G$	Commodity compatibility groups
$K$	Truck Types
$T(k)$	Trucks of type $k$ , $T(k) \subset S$ and $T(k_1) \cap T(k_2) = \phi$ , if $k_1 \neq k_2$
$V(g)$	Commodities not in compatibility group $g$ .

*Inputs-Parameters:*

$B$	Allocated budget or zero
$c(m)$	Bulk capacity of container $m$
$C(j)$	Number of quadcons vehicle $j$ can carry
$CCost(m)$	Cost of container $m$
$D(i)$	Demand for commodity $i$
$pw(i)$	Weight per pallet of commodity $i$ .
$Q_\beta$	$\beta$ -percentile of the standard normal density function
$qe(m)$	Bulk footprint of container $m$ (quadcon=1, bicon=2, 20' container=4)
$tw(m)$	Tare weight of container $m$
$U$	Maximum number of different truck types permitted
$VCost(j)$	Cost of vehicle $j$
$w(m)$	Payload of container $m$
$W(j)$	Payload of vehicle $j$
$\beta_o$	Probability of satisfying constraint associated with objective function
$\beta_c$	Probability of satisfying demand
$\gamma(j, m)$	1 if container $m$ cannot be transported by vehicle $j$ ; 0 otherwise
$\theta(s, r)$	1 if trailer $r$ cannot be towed by truck $s$ ; 0 otherwise

*Decision Variables:*

$am_{i,m}$	Amount of commodity $i$ in container $m$ , real number
$b_{i,m}$	Number of pallets loaded with commodity $i$ in container $m$ , integer
$d_{j,m}$	1 if container $m$ is transported using vehicle $j$ ; 0 otherwise
$gm_{m,g}$	1 if container $m$ is carrying commodities in group $g$ ; 0 otherwise
$h_k$	1 if vehicles of type $k$ are used; 0 otherwise
$Q_{s,r}$	1 if truck $s$ tows trailer $r$ , 0 otherwise
$VarR_{\beta_o}$	Value at Risk at level $\beta_o$
$x_j$	1 if vehicle $j$ is used; 0 otherwise
$z_m$	1 if container $m$ is used; 0 otherwise

## B1 The Objective Function

The objective function is simply the sum of the cost of the vehicles and containers selected in the scenario. This is represented algebraically as:

$$\min : \sum_{j \in J} VCost(j) \cdot x_j + \sum_{m \in M} CCost(m) \cdot z_m . \quad (B1)$$

## B2 The Model Constraints

The constraints for the deterministic variant of the mathematical programming model for the problem are summarized as follows:

$$\sum_{m \in M} am_{i,m} \geq D(i) \quad \forall i \in I \quad (B2)$$

$$\sum_{s \in S} \sum_{r \in R} \theta(s,r) \cdot Q(s,r) = 0 \quad (B3)$$

$$\sum_{j \in J} \sum_{m \in M} \gamma(j,m) \cdot d_{j,m} = 0 \quad (B4)$$

$$am_{i,m} - pw(i) \cdot b_{i,m} \leq 0 \quad \forall m \in M \quad (B5)$$

$$pw(i) \cdot b_{i,m} - am_{i,m} \leq pw(i) \quad \forall m \in M \quad (B6)$$

$$\sum_{i \in I} b_{i,m} - c(m) \cdot z_m \leq 0 \quad \forall m \in M \quad (B7)$$

$$\sum_{m \in M} qe(m) \cdot d_{j,m} = C(j) \cdot x_j \quad \forall j \in J \quad (B8)$$

$$\sum_{i \in I} am_{i,m} \leq w(m) \cdot z_m \quad \forall m \in M \quad (B9)$$

$$\sum_{i \in I} am_{i,m} + tw(m) \cdot z_m - \sum_{j \in J} \frac{qe(m) \cdot W(j)}{4} \cdot d_{j,m} \leq 0 \quad \forall m \in M \quad (B10)$$

$$\sum_{s \in S} Q_{s,r} - x_r = 0 \quad \forall r \in R \quad (B11)$$

$$\sum_{r \in R} Q_{s,r} - x_s \leq 0 \quad \forall s \in S \quad (B12)$$

$$\sum_{j \in J} d_{j,m} - z_m = 0 \quad \forall m \in M \quad (\text{B13})$$

$$\sum_{i \in V'(g)} b_{i,m} + c(m) \cdot gm_{m,g} \leq c(m) \quad \forall m \in M \quad (\text{B14})$$

$$\sum_{i \in V(g)} b_{i,m} - c(m) \cdot gm_{m,g} \leq 0 \quad \forall m \in M \quad (\text{B15})$$

$$gm_{m,g} - \sum_{i \in V(g)} b_{i,m} \leq 0 \quad \forall m \in M \quad (\text{B16})$$

$$\sum_{k \in K} h_k \leq U \quad (\text{B17})$$

$$h_k - \sum_{j \in T(k)} x_j \leq 0 \quad \forall k \in K \quad (\text{B18})$$

$$\sum_{j \in T(k)} x_j - |T(k)| \cdot h_k \leq 0 \quad \forall k \in K \quad (\text{B19})$$

$$am_{i,m} \geq 0 \quad (\text{B20})$$

$$gm_{m,g}, d_{j,m}, x_j, z_m, h_k \in \{0, 1\}. \quad (\text{B21})$$

Equation B2 requires the amount of a commodity delivered to be more than or equal to the demand. Equations B3 and B4 are compatibility constraints. They prevent certain truck-trailer (equation B3) and vehicle-container (B4) combinations.

There should be enough pallets to load the commodities to be transported (equation B5) but the number must be no more than the minimum required (equation B6). The bulk capacities of the containers (equation B7) and vehicles (equation B8) may not be exceeded. Also the total weight of commodities transported in each container should not exceed its payload (equation B9). Equation B10 ensures that vehicle payloads are not exceeded. This assumes that a vehicle's payload is distributed amongst the containers it transports in proportion to their bulk footprints. For example, a vehicle with a payload of 10 tonnes, may transport two quadcons and a bicon with the quadcons each weighing a maximum of two and a half tonnes and the bicon weighing a maximum of five tonnes.

Equations B11 and B12 ensure that each trailer is towed by at most one truck and equation B13 ensures that a container is loaded on only one vehicle if it is being used. Equations B14-B16 are commodity compatibility constraints. Only commodities in the same compatibility group may be loaded in the same container.

Equation B17 restricts the number of different types of trucks in the fleet. Equations B18 and B19 ensure that at least one truck of a type selected is included in the fleet. As a corollary, no truck of a type not selected is included in the fleet.

### **B3 The Deterministic Equivalent of the Fleet Mix CCGP Model**

Since it is assumed that the cost function coefficients ( $VCost(j)$  and  $CCost(m)$ ) and demand ( $D(i)$ ) are uncertain, equations B1 and B2 may be converted to chance constraints as described in Section A2. Consequently, the deterministic equivalent model of the CCGP variant of the mixed integer programming model presented above is given as:

$$\min \quad VaR_{\beta_o} \tag{B22}$$

subject to:

$$\mathbb{E}[f(x_j, z_m)] - B + Q_{\beta_o} \sqrt{\mathbb{V}[f(x_j, z_m)]} \leq VaR_{\beta_o} \tag{B23}$$

$$\mathbb{E}[D(i)] - \sum_{m \in M} a_{i,m} + Q_{\beta_c} \sqrt{\mathbb{V}[D(i)]} \leq 0 \quad \forall i \in I \tag{B24}$$

equation B3

equation B4

⋮

equation B21

$$VaR_{\beta_o} > 0 ,$$

with  $f(x_j, z_m) = \sum_{j \in J} VCost(j) \cdot x_j + \sum_{m \in M} CCost(m) \cdot z_m$ .

In the model above, equation B1 has been converted into a constraint (equation B23) with the introduction of a new objective (equation B22), and equation B2 has been replaced with equation B24.

The  $B$  in equation B23 is the objective function goal or target. If this value is set to zero then  $VaR_{\beta_o}$  represents the required budget which has a  $100 \times \beta_o\%$  chance of not being exceeded. On the other hand, if  $B$  is set to the allocated budget, then there is a  $100 \times \beta_o\%$  chance that the cost overrun would be less than  $VaR_{\beta_o}$ . In the next section, results from the application of the model to the problem described in the scenario are presented.



**Annex C**  
**LIST OF ABBREVIATIONS, ACRONYMS AND MATHEMATICAL NOTATIONS**

*Abbreviations and Acronyms:*

AST	Acquisition Support Team
CCGP	Chance-Constrained Goal Programming
CCP	Chance-Constrained Programming
CORA	Centre for Operational Research and Analysis
CVaR	Conditional Value at Risk
DMGOR	Directorate Materiel Group Operational Research
DRDC	Defence Research and Development Canada
ECP	Extended Cutting Plane
ft	Foot (or Feet)
GP	Goal Programming
IMP	Individual Meal Pack
kg	kilograms
LHS	Load Handling System
LVMP	Logistic Vehicle Modernization Project
MSVS	Medium Support Vehicle System
NATO	North Atlantic Treaty Organization
NEOS	Network-Enabled Optimization System
PLS	Palletized Load System
POL	Petroleum Oils and Lubricants
VaR	Value at Risk
VRP	Vehicle Routing Problem

*Mathematical Notations*

$\mathbb{E}[\cdot]$	Expectation
$\forall$	For all
$\in$	Is a member of
$\subset$	Is a subset of
$ \cdot $	Set cardinality
$\cup$	Set union
$\cap$	Set intersection
$\mathbb{V}[\cdot]$	Variance

